



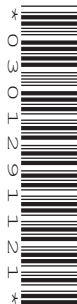
Cambridge International AS & A Level

CANDIDATE
NAMECENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--



FURTHER MATHEMATICS

9231/21

Paper 2 Further Pure Mathematics 2

May/June 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



- 1 Find the roots of the equation $z^3 = 27 - 27i$, giving your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi \leq \theta < \pi$. [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



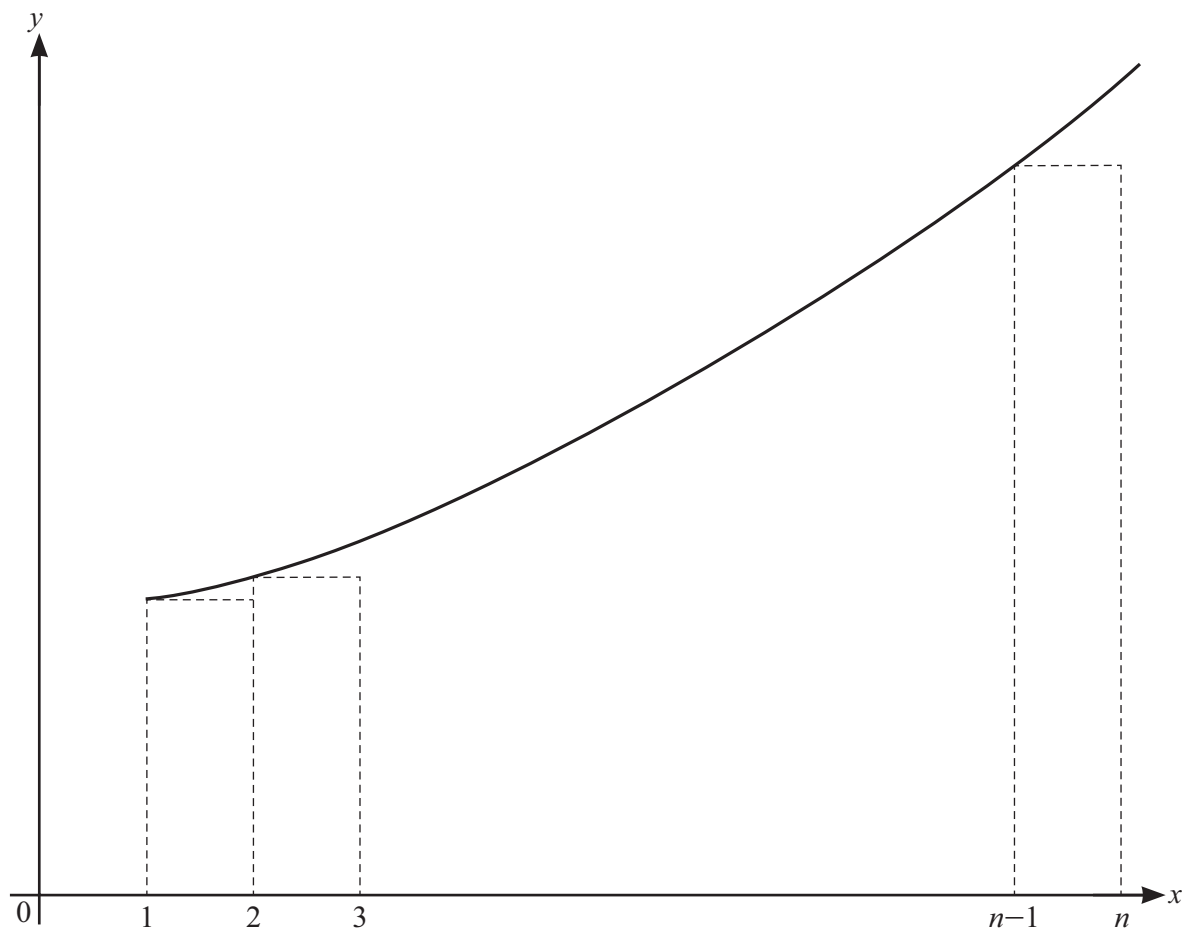
* 0000800000005 *



5

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

4



The diagram shows the curve with equation $y = \frac{1}{\sqrt{x}}e^{\sqrt{x}}$ for $x \geq 1$, together with a set of $n-1$ rectangles of unit width.

(a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^n \frac{1}{\sqrt{r}} e^{\sqrt{r}} < \left(2 + \frac{1}{\sqrt{n}}\right) e^{\sqrt{n}} - 2e. \quad [5]$$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





- (b) Use a similar method to find, in terms of n , a lower bound for $\sum_{r=1}^n \frac{1}{\sqrt{r}} e^{\sqrt{r}}$. [4]



[illegible]



- 6 (a) Starting from the definitions of $\tanh$ and sech in terms of exponentials, prove that

$$1 - \tanh^2 u = \operatorname{sech}^2 u. \quad [3]$$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Show that $\frac{d}{dt}(\operatorname{sech}^{-1} t) = -\frac{1}{t\sqrt{1-t^2}}.$ [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





It is given that

$$x = \tanh^{-1} t \quad \text{and} \quad y = t \operatorname{sech}^{-1} t, \quad \text{for } 0 < t < 1.$$

- (c) Show that $\frac{dy}{dx} = -\sqrt{1-t^2} + (1-t^2)\operatorname{sech}^{-1} t$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

- (d) Find $\frac{d^2 y}{dx^2}$ in terms of t . [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





7 Find the solution of the differential equation

$$\frac{dy}{dx} - \frac{x+5}{x^2+10x+61}y = 1,$$

given that $y = 0$ when $x = 3$. Give your answer in an exact form.

[10]

[illegible]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



- 8 (a) It is given that λ is an eigenvalue of the non-singular square matrix $\mathbf{A}$, with corresponding eigenvector $\mathbf{e}$.

Show that $\mathbf{e}$ is an eigenvector of $\mathbf{A}^3$ with corresponding eigenvalue λ^3 . [2]

.....

.....

.....

.....

.....

.....

.....

.....

The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} -1 & 3 & 4 \\ 0 & 1 & 0 \\ 0 & -2 & 5 \end{pmatrix}.$$

- (b) Show that the eigenvalues of $\mathbf{A}$ are -1 , 1 and 5 . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





- (c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A} - 2\mathbf{I} = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$. [6]

[illegible]

- (d) Use the characteristic equation of $\mathbf{A}$ to show that $(\mathbf{A} - 2\mathbf{I})^3 = a\mathbf{A}^2 + b\mathbf{A} + c\mathbf{I}$ where a , b and c are constants to be determined. [3]

This image shows a blank sheet of white paper with ten horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There is no text or other markings on the paper.



If you use the following page to complete the answer to any question, the question number must be clearly shown.

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

* 0000800000017 *



17

BLANK PAGE

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN



* 0000800000018 *



18

BLANK PAGE



* 0000800000019 *



BLANK PAGE

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN





Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

